

Coherent states, fractals and brain waves

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I show that a functional representation of self-similarity (as the one occurring in fractals) is provided by squeezed coherent states. In this way, the dissipative model of brain is shown to account for the self-similarity in brain background activity suggested by power-law distributions of power spectral densities of electrocorticograms. I also briefly discuss the action-perception cycle in the dissipative model with reference to intentionality in terms of trajectories in the memory state space.

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I. INTRODUCTION

The first time I met Walter Freeman was in 2000, at one of those crowded conferences with many parallel sessions and just few (very few!) interesting plenary talks. One of these was indeed the talk by Lotfi Zadeh, another one was by Walter. My interest in nonlinear dynamical systems and my curiosity in watching through the fence what is going on in the neighboring garden brought me to that conference. I was surprised by myself since I felt to understand a good part of those two talks. Usually this does not happen to me. In particular, Walter was talking of his research in mesoscopic brain dynamics with a language very near to the one familiar to a physicist trained in the study of the formation of ordered patterns in condensed matter physics and in high energy physics. Of course, he was not using the machinery of quantum field theory which is the one used by physicists in their study of many-body systems. However, he explained in clear words his laboratory observations and his theoretical analysis showing that the mesoscopic neural activity of neocortex appears to consist of the dynamical formation of spatially extended neuronal domains in which widespread cooperation supports brief epochs of patterned synchronized oscillations [1]. This vivid physical picture of the brain mesoscopic activity was confirming to me what Hiroomi Umezawa, one of the fathers of modern quantum field theory, was meaning by saying that [2]: "In any material in condensed matter physics any particular information is carried by certain ordered pattern maintained by certain long range correlation mediated by massless quanta. It looked to me that this is the only way to memorize some information; memory is a printed pattern of order supported by long range correlations". The key words here are "widespread cooperation" supporting "patterned synchronized oscillations" and "ordered pattern maintained by certain long range correlation". In 1967, when he was still in Naples, Umezawa proposed indeed a model of brain dynamics based on the dynamical generation of such long range correlation in the way they are normally treated in many-body theory [3]. As a matter of fact, there is no alternative formalism to explain the *dynamical* generation of ordered patterns in solid state physics and elementary particle physics. Crystals, magnets, and other ordered patterns observed at reasonably high temperature, superconductors at much lower temperature, the lowest energy state, usually called the "vacuum", in the standard model of particle physics, are in fact all successfully described by many-body physics with an incredible precision in the prediction of measured quantities. The success of such a quantum field theory (QFT) is really impressive and led Umezawa to say [2]: "In any case soon after I moved to Naples, I strongly felt that there should be a long range correlation which controls the brain function. If I could know what kind of correlation, I would be able to write down the Hamiltonian, bringing the brain science to the level of condensed matter physics." Of course, Umezawa was a physicist and long range correlation modes were a familiar tool to work with. However, already in the early 1940s Lashley's work showing the diffuse, non-local nature of brain activity, induced him to talk of "mass action" in the storage and retrieval of memories in the brain, and he observed: "...Here is the dilemma. Nerve impulses are transmitted ...from cell to cell through definite intercellular connections. Yet, all behavior seems to be determined by masses of excitation...within general fields of activity, without regard to particular nerve cells... What sort of nervous organization might be capable of responding to a pattern of excitation without limited specialized path of conduction? The problem is almost universal in the activity of the nervous system" (pp. 302–306 of Ref. 4). Umezawa also knew about the work by Pribram who, soon after the discovery of the laser light in the early 1960s [5, 6] following the theoretical studies by Gabor [7, 8], proposed his holographic hypothesis [9]. The interesting point in Umezawa's many-body model for the brain is that two main ingredients appear there together: the notion of "field" introduced by Lashley in his puzzling dilemma and the notion of "coherence", intrinsic to the laser theory inspiring Pribram view. Both these notions are basic ones in the QFT dynamics generating ordered patterns, but not in neuroscience, and in general in biology and biochemistry, where the atomistic view of assembling little pieces together has been

prevailing on the search of the microscopic *dynamical* laws ruling their cooperative behavior so that the mesoscopic and macroscopic functioning of the system could emerge. One must have the courage of a Lashley and of a Pribram to dare to introduce the field concept and the wave notion of coherence. This is why, when listening Walter Freeman talking of dynamical widespread cooperation supporting patterned synchronized oscillations, it was clear to me that he is one of those few people who dare to open new paths in the forest.

Today, with the advent of advanced technologies, the amplitude modulated (AM) patterns and the phase modulated (PM) patterns in the brain are of common observational access and it becomes imperative to understand how out of the behavior of the single neuronal cells a *transition* may occur into the coherent behavior of a collective neuronal assembly, or in Lashley's words, "what sort of nervous organization might be capable of responding to a pattern of excitation". Laboratory observations of the brain functioning show that the observed cortical collective activity cannot be fully accounted for by the electric field of the extracellular dendritic current or the extracellular magnetic field from the high-density electric current inside the dendritic shafts, which are much too weak, or by the chemical diffusion, which is much too slow [10, 11, 12]. Patterns of phase-locked oscillations are intermittently present in resting, awake subjects as well as in the same subject actively engaged in cognitive tasks requiring interaction with environment, so they are best described as properties of the background activity of brains that is modulated upon engagement with the surround. These "packets of waves" extend over spatial domains covering much of the hemisphere in rabbits and cats [13, 14, 15, 16], and over regions of linear size of about 19 cm [17] in human cortex with near zero phase dispersion [18, 19]. Synchronized oscillation of large-scale neuronal assemblies in β and γ ranges have been detected also by magnetoencephalographic (MEG) imaging in the resting state and in motor task-related states of the human brain [20]. The AM patterns turn out to be the result of training the subjects to recognize stimuli under reinforcement and respond to them appropriately. The patterns are shaped by modifications of synaptic strengths during training. The formation of a Hebbian nerve cell assembly thus occurs for each learned category of stimulus.

Umezawa's many-body model [3] and its extension to dissipative dynamics, the dissipative many-body model of brain [21] are based on the QFT notion of spontaneous breakdown of symmetry (SBS). I want to stress that in such models the neuron and the glia cells are treated as classically behaving systems. The quantum degrees of freedom are related with the symmetry of the dynamics. When such a symmetry is not the symmetry of the least energy state (the ground state or the vacuum) of the system, the dynamical symmetry is said to be spontaneously broken and mathematical consistency requires the existence of massless particles. These are called the Nambu-Goldstone (NG) quanta or modes [22]. They are boson particles normally observed in solid state physics. Examples are the phonon modes in the crystals, the magnon modes in ferromagnets, etc. They can be described, as customary in a quantum theory, as the quanta associated to certain waves, such as the elastic wave in crystals, the spin wave in ferromagnets. The role of such waves is the one of establishing long range correlation among the system constituents. For example, in magnets, the elementary magnetic dipoles are forced to oscillate "in phase" under the correlation established by the spin waves, i.e. by the magnon quanta spanning the extended domain which thus gets characterized by its macroscopic magnetization. We thus see that the mechanism of spontaneous breakdown of symmetry is at the origin of the *change of scale*: from the microscopic scale of the elementary constituent dynamics to the macroscopic scale of the system magnetization, which is therefore a measure of the ordering of the elementary constituents and is for that reason called the "order parameter". An essential point is that the generation of the ordering depends on the inner dynamics of the system, not on the strength or on other properties of the external stimulus causing the breakdown of the symmetry. The external field (the trigger or stimulus) is thus responsible of the "phase transition" from the normal (zero magnetization) phase to the magnetic phase. We thus learn that the mathematical structure of the theory must be adequate to allow physically distinct phases (technically called unitarily inequivalent representations of the quantum algebra). QFT possesses indeed such a mathematical structure. The ground states corresponding to physically distinct phases are characterized by distinct degrees of ordering which are described by different numbers of NG modes *condensed* in them. Such a condensation of NG modes in the ground states is a *coherent* condensation, which physically expresses the "in phase", i.e. synchronized, dipole oscillation. In quantum mechanics (QM), on the contrary, all the state representations are physically equivalent (unitarily equivalent) and therefore QM is not useful to describe phase transitions [23, 24].

The quantum variables relevant to the many-body model have been identified in subsequent developments [21, 25] as the electric dipole vibrational modes of the water molecules which constitutes the matrix in which neurons and glia cells and other mesoscopic units are embedded. The spontaneous breakdown of the rotational symmetry of electrical dipoles of water and other molecules [21, 25, 26] implies the existence of NG modes which in such a context have been called the dipole wave quanta (DWQ). The system ground state is obtained in terms of coherent condensation of the DWQ. As suggested by the well known electrical properties of cell membranes and by the experimental observations [27, 28] of slow fluctuations in neuronal membrane polarization (the so called up and down states) corresponding to that of spontaneous fluctuations in the fMRI signal, the electrical dipole oscillatory matrix in which the neuronal electrophysical and electrochemical activity is embedded [29] cannot be ignored, indeed.

In the many-body model memory is obtained as an ordered pattern produced by the coherent condensation of the

DWQ under the external stimulus acting as a trigger for the symmetry breakdown. The DWQ are the agents by which the coordination emerges. The “memory state” is therefore a coherent state for the basic quantum variables, whose mesoscopic order parameter expresses, at the synaptic level, the amplitude and phase modulation of the carrier signal.

The original model suffers, however, of a very much limited memory capacity. Any subsequent stimulus, producing the associated DWQ condensation, cancels the one produced by the preceding stimulus (overprinting). It has been shown [21] that such a problem may find a solution when the model is modified so to include one of the intrinsic properties of brain system, the one of being an open system ruled, therefore, by a dissipative dynamics. The dissipative many-body model of brain predicts then the coexistence of physically distinct amplitude modulated and phase modulated patterns correlated with categories of conditioned stimuli, and the remarkably rapid onset of AM patterns into irreversible sequences that resemble cinematographic frames. These are indeed two main features of neurophysiological data [11, 12, 30] fitted by the dissipative model of brain. In this paper I will not insist further in the description of the dissipative many-body model, which can be found in the existing literature [11, 12, 21, 30, 31, 32, 33, 34]. I will instead consider only two features of the model. One is related with the scale free and self-similarity property of the brain dynamics as suggested from power-law distributions of power spectral densities of electrocorticograms (ECoGs). This is discussed in Section 2. The other one, discussed in Section 3, is related with the brain-environment complex coupling. Conclusive remarks are reported in Section 4.

II. SELF-SIMILARITY AND COHERENT STATES

Observation of the brain background activity shows that self-similarity characterizes the brain ground state. Indeed, measurements of the durations, recurrence intervals and diameters of neocortical EEG phase patterns have power-law distributions with no detectable minima. The power spectral densities in both time and space of ECoGs from surface arrays conform to power-law distributions [13, 14, 35, 36, 37], which suggests that the activity patterns generated by neocortical neuropil might be scale-free [38, 39] with self-similarity in ECoGs patterns over distances ranging from hypercolumns to an entire cerebral hemisphere (this might explain the similarity of neocortical dynamics in mammals differing in brain size by 4 orders of magnitude, from mouse [40] to whale [41], which contrasts strikingly with the relatively small range of size of avian, reptilian and dinosaur brains lacking neocortex) [11].

In the dissipative many-body model, one of the main actors is the (brain) ground state, which, at some initial time t_0 , may be represented as the collection (or the superposition) of a full set of ground states, denote them by $|0\rangle_{\mathcal{N}}$, for all \mathcal{N} . \mathcal{N} denotes the set of integers \mathcal{N}_{a_κ} and $\mathcal{N}_{\tilde{a}_\kappa}$ defining the “initial value” of the condensate, $\mathcal{N} \equiv \{\mathcal{N}_{a_\kappa} = \mathcal{N}_{\tilde{a}_\kappa}, \text{ for all } \kappa\}$, at $t_0 = 0$, and defines the *order parameter* associated with the information recorded at time $t_0 = 0$. \mathcal{N}_{a_κ} and $\mathcal{N}_{\tilde{a}_\kappa}$ denote the number of DWQ a_κ and \tilde{a}_κ condensed in the state $|0\rangle_{\mathcal{N}}$. The label κ generically denotes degrees of freedom such as, e.g., spatial momentum, etc. The \tilde{a}_κ operators represents the system’s environment; this behaves as the time-reversed image of the dissipative system since the energy flux *in*-coming into it is obtained by time-reversal of the energy flux *out*-going from the dissipative system, and vice-versa. The environment \tilde{a}_κ operators are thus time-reversed mirror modes of the brain system. Their introduction is necessary in order to set up the formalism for the dissipative system [21, 42, 43]. In such a formalism the environment is thus described as the *time-reversed copy*, the *Double* [34], of the dissipative system. Such a *doubling* of the degrees of freedom is mathematically well defined by the coproduct mapping in the deformed Hopf algebra. Here, however, I will not insist on mathematical details which can be found in the literature [44, 45].

States $|0\rangle_{\mathcal{N}}$ and $|0\rangle_{\mathcal{N}'}$, for $\mathcal{N} \neq \mathcal{N}'$, are non-overlapping (unitarily inequivalent) states in the infinite volume limit: ${}_{\mathcal{N}}\langle 0|0\rangle_{\mathcal{N}'} \rightarrow 0$ as $V \rightarrow \infty$, for all $\mathcal{N}, \mathcal{N}'$, $\mathcal{N} \neq \mathcal{N}'$. The brain may occupy any of the ground states $|0\rangle_{\mathcal{N}}$, or it may be in any state that is a collection or superposition of these brain-environment ground states. The state $|0(t)\rangle_{\mathcal{N}}$ denotes the state $|0\rangle_{\mathcal{N}}$ at time t specified by the initial value \mathcal{N} , at $t_0 = 0$. $|0(t)\rangle_{\mathcal{N}}$ is recognized to be, at any time t , a finite temperature state and it can be shown to be a squeezed coherent state [21, 46, 47] (see below for the definition of squeezing). In this Section I will show that a functional representation of self-similarity (as the one occurring in fractals) is provided by squeezed coherent states. In this way we see that the dissipative model of brain accounts for the self-similarity in brain background activity.

Below I will limit my considerations to the self-similarity property of fractals which are generated iteratively according to a prescribed recipe, the so-called deterministic fractals (fractals generated by means of a random process, called “random fractals” [48], will not be considered here. For a discussion on this point and a relation of fractals with brain/mind see also Ref. 49. In some sense, the self-similarity property is the *most important property* of fractals (p. 150 in Ref. 50). The conjecture that a relation between fractals and the algebra of coherent states exists was presented in Ref. 51. In the following I will closely follow the presentation of Ref. 52.

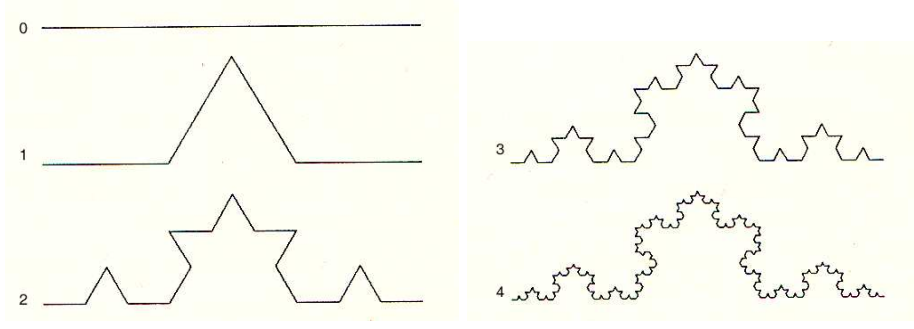


FIG. 1: The first five stages of Koch curve.

A. Self-similarity and the coherent state algebra

I consider for simplicity the *Koch curve* (Fig. 1). The discussion can be extended to other iteratively constructed fractals. Consider a one-dimensional, $d = 1$, segment u_0 of unit length L_0 , called the *initiator* [48]. As usual, this is called the step, or stage, of order $n = 0$. The length L_0 is then divided by the reducing factor $s = 3$, and the rescaled unit length $L_1 = \frac{1}{3}L_0$ is adopted to construct the new “deformed segment” u_1 made of $\alpha = 4$ units L_1 (step of order $n = 1$). u_1 is called the *generator* [48]. Note that such a “deformation” of the u_0 segment is only possible provided one “gets out” of the one dimensional straight line r to which the u_0 segment belongs: this means that in order to construct the u_1 segment “shape” the one dimensional constraint $d = 1$ has to be relaxed: the shape, made of $\alpha = 4$ units L_1 , lives in some $d \neq 1$ dimensions. Thus we write $u_{1,q}(\alpha) \equiv q \alpha u_0$, $q = \frac{1}{3^d}$, $d \neq 1$, where d has to be determined.

Denoting by $\mathcal{H}(L_0)$ lengths, surfaces or volumes, the familiar scaling law

$$\mathcal{H}(\lambda L_0) = \lambda^d \mathcal{H}(L_0) , \quad (1)$$

holds when lengths are (homogeneously) scaled, $L_0 \rightarrow \lambda L_0$. A square S whose side is L_0 scales to $\frac{1}{2^2}S$ when $L_0 \rightarrow \lambda L_0$ with $\lambda = \frac{1}{2}$. A cube V of same side with same rescaling of L_0 scales to $\frac{1}{2^3}V$. Thus $d = 2$ and $d = 3$ for surfaces and volumes, respectively. Note that $\frac{S(\frac{1}{2}L_0)}{S(L_0)} = p = \frac{1}{4}$ and $\frac{V(\frac{1}{2}L_0)}{V(L_0)} = p = \frac{1}{8}$, respectively, so that in both cases $p = \lambda^d$. For the length L_0 , it is $p = \frac{1}{2} = \frac{1}{2^d} = \lambda^d$ and of course $d = 1$.

By generalizing and extending this to the case of any other “ipervolume” \mathcal{H} one considers thus the ratio

$$\frac{\mathcal{H}(\lambda L_0)}{\mathcal{H}(L_0)} = p , \quad (2)$$

and assuming that Eq. (1) is still valid “by definition”, one obtains

$$p \mathcal{H}(L_0) = \lambda^d \mathcal{H}(L_0) , \quad (3)$$

i.e. $p = \lambda^d$. For the Koch curve, setting $\alpha = \frac{1}{p} = 4$ and $q = \lambda^d = \frac{1}{3^d}$, $p = \lambda^d$ gives

$$q\alpha = 1 , \quad \text{where} \quad \alpha = 4, \quad q = \frac{1}{3^d} , \quad (4)$$

i.e.

$$d = \frac{\ln 4}{\ln 3} \approx 1.2619 . \quad (5)$$

d is called the *fractal dimension*, or the *self-similarity dimension* [50].

With reference to the Koch curve, I observe that the meaning of Eq. (3) is that in the “deformed space”, to which $u_{1,q}$ belongs, the set of four segments of which $u_{1,q}$ is made “equals” (is equivalent to) the three segments of which u_0 is made in the original “undeformed space”. The (fractal) dimension d is the dimension of the deformed space that

ensures the existence of a solution of the relation $\frac{1}{\alpha} = \frac{1}{4} = \frac{1}{3^d} = q$, which for $d = 1$ would be trivially wrong. In this sense d is a measure of the “deformation” of the $u_{1,q}$ -space *with respect to the u_0 -space*. In other words, we require that the measure of the deformed segment $u_{1,q}$ with respect to the undeformed segment u_0 be 1: $\frac{u_{1,q}}{u_0} = 1$, namely $\alpha q = \frac{4}{3^d} = 1$. In the following, for brevity I will thus set $u_0 = 1$, whenever no misunderstanding arises.

Since the deformation of u_0 into $u_{1,q}$ is performed by varying the number α of rescaled unit segments from 3 to 4, we expect that α and its derivative $\frac{d}{d\alpha}$ play a relevant role in the fractal structure. We will see indeed that $(\alpha, \frac{d}{d\alpha})$ play the role of conjugate variables (cf. Eq. (10)).

Steps of higher order n , $n = 2, 3, 4, \dots$, can be obtained by iteration of the deformation process keeping $q = \frac{1}{3^d}$ and $\alpha = 4$. For the n th order deformation we have

$$u_{n,q}(\alpha) \equiv (q\alpha) u_{n-1,q}(\alpha), \quad n = 1, 2, 3, \dots \quad (6)$$

i.e., for any $n \in \mathcal{N}_+$

$$u_{n,q}(\alpha) = (q\alpha)^n u_0. \quad (7)$$

By proceeding by iteration, or, equivalently, by requiring that $\frac{u_{n,q}(\alpha)}{u_0}$ be 1 for any n , gives $(q\alpha)^n = 1$ and Eq. (5) is again obtained. Notice that the fractal is mathematically defined in the limit $n \rightarrow \infty$ of the deformation process. The definition of fractal dimension is indeed given starting from $(q\alpha)^n = 1$ in the $n \rightarrow \infty$ limit. Since $L_n \rightarrow 0$ for $n \rightarrow \infty$, the Koch fractal is a curve which is everywhere non-differentiable [50].

Eqs. (6) and (7) express in analytic form, *in the $n \rightarrow \infty$ limit*, the *self-similarity* property of a large class of fractals (the Sierpinski gasket and carpet, the Cantor set, etc.): “cutting a piece of a fractal and magnifying it isotropically to the size of the original, both the original and the magnification look the same” [48]. In this sense one also says that fractals are “scale free”, namely viewing a picture of part of a fractal one cannot deduce its actual size if the unit of measure is not given in the same picture [53]. I stress that only in the $n \rightarrow \infty$ limit self-similarity is defined (self-similarity does not hold when considering only a finite number n of iterations).

I recall that invariance, in the limit of $n \rightarrow \infty$ iterations, only under anisotropic magnification is called self-affinity. The discussion below can be extended to self-affine fractals. I will not discuss here the measure of lengths in fractals, the Hausdorff measure, the fractal “mass” and other fractal properties. The reader is referred to the existing literature.

My main observation is now that Eq. (7) expresses the deep formal connection with the theory of coherent states and the related algebraic structure. In order to make such a connection explicit, I observe that, by considering in full generality the complex α -plane, the functions

$$u_n(\alpha) = \frac{\alpha^n}{\sqrt{n!}}, \quad u_0(\alpha) = 1, \quad n \in \mathcal{N}_+, \quad \alpha \in \mathcal{C}, \quad (8)$$

form in the space \mathcal{F} of the entire analytic functions a basis which is orthonormal under the gaussian measure $d\mu(\alpha) = \frac{1}{\pi} e^{-|\alpha|^2} d\alpha d\bar{\alpha}$. In Eq. (8) the factor $\frac{1}{\sqrt{n!}}$ ensures the normalization condition with respect to the gaussian measure. In the following it is always $n \in \mathcal{N}_+$.

The functions $u_{n,q}(\alpha)|_{q \rightarrow 1}$ in Eq. (7) are thus immediately recognized to be nothing but the restriction to real α of the functions in Eq. (8), apart the normalization factor $\frac{1}{\sqrt{n!}}$. The study of the fractal properties may be thus carried on in the space \mathcal{F} of the entire analytic functions, by restricting, at the end, the conclusions to real α , $\alpha \rightarrow \text{Re}(\alpha)$. Furthermore, since actually in Eq. (7) it is $q \neq 1$ ($q < 1$), one also needs to consider the “ q -deformed” algebraic structure of which the space \mathcal{F} provides a representation.

The space \mathcal{F} is a vector space which provides the so called Fock-Bargmann representation (FBR) [47, 54] of the Weyl–Heisenberg algebra generated by the set of operators $\{a, a^\dagger, \mathbf{1}\}$:

$$[a, a^\dagger] = \mathbf{1}, \quad [N, a^\dagger] = a^\dagger, \quad [N, a] = -a, \quad (9)$$

where $N \equiv a^\dagger a$, with the identification:

$$N \rightarrow \alpha \frac{d}{d\alpha}, \quad a^\dagger \rightarrow \alpha, \quad a \rightarrow \frac{d}{d\alpha}. \quad (10)$$

The $u_n(\alpha)$ (Eq. (8)) are eigenkets of N with integer (positive and zero) eigenvalues. The FBR is the Hilbert space \mathcal{K} generated by the $u_n(\alpha)$, i.e. the whole space \mathcal{F} of entire analytic functions. Any vector $|\psi\rangle$ in \mathcal{K} is associated, in a one-to-one correspondence, with a function $\psi(\alpha) \in \mathcal{F}$ and is thus described by the set $\{c_n; c_n \in \mathcal{C}, \sum_{n=0}^{\infty} |c_n|^2 = 1\}$

defined by its expansion in the complete orthonormal set of eigenkets $\{|n\rangle\}$ of N :

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle \rightarrow \psi(\alpha) = \sum_{n=0}^{\infty} c_n u_n(\alpha), \quad (11)$$

$$\langle\psi|\psi\rangle = \sum_{n=0}^{\infty} |c_n|^2 = \int |\psi(\alpha)|^2 d\mu(\alpha) = \|\psi\|^2 = 1, \quad (12)$$

$$|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle, \quad (13)$$

where $|0\rangle$ denotes the vacuum vector, $a|0\rangle = 0$, $\langle 0|0\rangle = 1$. The series expressing $\psi(\alpha)$ in Eq. (11) converges uniformly in any compact domain of the α -plane due to the condition $\sum_{n=0}^{\infty} |c_n|^2 = 1$ (cf. Eq. (12)), confirming that $\psi(\alpha)$ is an entire analytic function.

The Fock–Bargmann representation provides a simple frame to describe the usual coherent states (CS) [6, 47] $|\alpha\rangle$:

$$|\alpha\rangle = \mathcal{D}(\alpha)|0\rangle, \quad a|\alpha\rangle = \alpha|\alpha\rangle, \quad \alpha \in \mathcal{C}, \quad (14)$$

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} u_n(\alpha) |n\rangle. \quad (15)$$

The unitary displacement operator $\mathcal{D}(\alpha)$ in (14) is given by:

$$\mathcal{D}(\alpha) = \exp(\alpha a^\dagger - \bar{\alpha} a) = \exp\left(-\frac{|\alpha|^2}{2}\right) \exp(\alpha a^\dagger) \exp(-\bar{\alpha} a). \quad (16)$$

$\mathcal{D}(\alpha)$ is a bounded operator defined on the whole \mathcal{K} . It provides a representation of the Weyl–Heisenberg group usually denoted by W_1 [47]. We have

$$\mathcal{D}^{-1}(\alpha) a \mathcal{D}(\alpha) = a + \alpha. \quad (17)$$

The explicit relation between the CS and the entire analytic function basis $\{u_n(\alpha)\}$ is:

$$u_n(\alpha) = e^{\frac{1}{2}|\alpha|^2} \langle n|\alpha\rangle. \quad (18)$$

The set $\{|\alpha\rangle\}$ is an overcomplete set of states, from which, however, a complete set can be extracted. It is well known that in order to extract a complete set of CS from the overcomplete set it is necessary to introduce in the α -complex plane a regular lattice L , called the von Neumann lattice [47]. For a general discussion and original references see Ref. 47. See also Refs. 44 where the von Neumann lattice is discussed also in connection with the deformation of the Weyl–Heisenberg algebra introduced below.

I now introduce the finite difference operator \mathcal{D}_q defined by [45]:

$$\mathcal{D}_q f(\alpha) = \frac{f(q\alpha) - f(\alpha)}{(q-1)\alpha}, \quad (19)$$

with $f(\alpha) \in \mathcal{F}$, $q = e^\zeta$, $\zeta \in \mathcal{C}$. \mathcal{D}_q reduces to the standard derivative for $q \rightarrow 1$ ($\zeta \rightarrow 0$). In the space \mathcal{F} , \mathcal{D}_q satisfies, together with α and $\alpha \frac{d}{d\alpha}$, the commutation relations:

$$[\mathcal{D}_q, \alpha] = q^{\alpha \frac{d}{d\alpha}}, \quad \left[\alpha \frac{d}{d\alpha}, \mathcal{D}_q\right] = -\mathcal{D}_q, \quad \left[\alpha \frac{d}{d\alpha}, \alpha\right] = \alpha, \quad (20)$$

which, as for Eq. (10), lead us to the identification

$$N \rightarrow \alpha \frac{d}{d\alpha}, \quad \hat{a}_q \rightarrow \alpha, \quad a_q \rightarrow \mathcal{D}_q, \quad (21)$$

with $\hat{a}_q = \hat{a}_{q=1} = a^\dagger$ and $\lim_{q \rightarrow 1} a_q = a$ on \mathcal{F} . The algebra (20) is the q -deformation of the algebra (9). For shortness I omit to discuss further the properties of \mathcal{D}_q and the q -deformed algebra (20). More details can be found in Refs. 44. Here I only recall that the operator q^N acts on the whole \mathcal{F} as

$$q^N f(\alpha) = f(q\alpha), \quad f(\alpha) \in \mathcal{F}. \quad (22)$$

This result was originally obtained in Refs. 44, where it was realized that the q -deformation arises whenever one deals with some finite scale. A finite scale occurs indeed also in the present case of fractals and therefore also in this case we in fact have a deformation of the algebra.

Eq. (22) applied to the coherent state functional (15) gives

$$q^N |\alpha\rangle = |q\alpha\rangle = \exp\left(-\frac{|q\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{(q\alpha)^n}{\sqrt{n!}} |n\rangle, \quad (23)$$

and, since $q\alpha \in \mathcal{C}$, from Eq. (14),

$$a |q\alpha\rangle = q\alpha |q\alpha\rangle, \quad q\alpha \in \mathcal{C}. \quad (24)$$

By recalling that we have set $u_0 \equiv 1$, the n th fractal iteration, Eq. (7), is obtained by projecting out the n th component of $|q\alpha\rangle$ and restricting to real $q\alpha$, $q\alpha \rightarrow \text{Re}(q\alpha)$:

$$u_{n,q}(\alpha) = (q\alpha)^n = \sqrt{n!} \exp\left(\frac{|q\alpha|^2}{2}\right) \langle n|q\alpha\rangle, \quad q\alpha \rightarrow \text{Re}(q\alpha). \quad (25)$$

for any $n \in \mathcal{N}_+$. Taking into account that $\langle n| = \langle 0| \frac{(a)^n}{\sqrt{n!}}$, Eq. (25) gives

$$u_{n,q}(\alpha) = (q\alpha)^n = \exp\left(\frac{|q\alpha|^2}{2}\right) \langle 0|(a)^n |q\alpha\rangle, \quad n \in \mathcal{N}_+, \quad q\alpha \rightarrow \text{Re}(q\alpha), \quad (26)$$

which shows that the operator $(a)^n$ acts as a “magnifying” lens [48]: the n th iteration of the fractal can be “seen” by applying $(a)^n$ to $|q\alpha\rangle$ and restricting to real $q\alpha$:

$$\langle q\alpha|(a)^n |q\alpha\rangle = (q\alpha)^n = u_{n,q}(\alpha), \quad q\alpha \rightarrow \text{Re}(q\alpha). \quad (27)$$

Eq. (24) expresses the invariance of the coherent state representing the fractal under the action of the operator $\frac{1}{q\alpha}a$. This reminds us of the fixed point equation $W(A) = A$, where W is the Hutchinson operator [48], characterizing the iteration process for the fractal A in the $n \rightarrow \infty$ limit. Such an invariance property allows to consider the coherent functional $\psi(q\alpha)$ as an “attractor” in \mathcal{C} .

In conclusion, the operator q^N applied to $|\alpha\rangle$ “produces” the fractal in the functional form of the coherent state $|q\alpha\rangle$ (cf Eq. (23)). The n th fractal stage of iteration, $n = 0, 1, 2, \dots, \infty$ is represented, in a one-to-one correspondence, by the n th term in the coherent state series in Eq. (23). I call q^N *the fractal operator*.

Eqs. (25), (26) and (27) formally establish the searched connection between fractal self-similarity and the (q -deformed) algebra of the coherent states.

B. Self-similarity and squeezed coherent states

I now look at the fractal operator q^N from a different perspective and consider the identity

$$2\alpha \frac{d}{d\alpha} \psi(\alpha) = \left\{ \frac{1}{2} \left[\left(\alpha + \frac{d}{d\alpha} \right)^2 - \left(\alpha - \frac{d}{d\alpha} \right)^2 \right] - 1 \right\} \psi(\alpha), \quad (28)$$

which holds in the Hilbert space identified with the space \mathcal{F} of entire analytic functions $\psi(\alpha)$. It is convenient to set $\alpha \equiv x + iy$, x and y denoting the real and the imaginary part of α , respectively. I then introduce the operators

$$c = \frac{1}{\sqrt{2}} \left(\alpha + \frac{d}{d\alpha} \right), \quad c^\dagger = \frac{1}{\sqrt{2}} \left(\alpha - \frac{d}{d\alpha} \right), \quad [c, c^\dagger] = \mathbf{1}. \quad (29)$$

In \mathcal{F} , c^\dagger is the conjugate of c [44, 47]. In the limit $\alpha \rightarrow \text{Re}(\alpha)$, i.e. $y \rightarrow 0$, c and c^\dagger turn into the conventional annihilation and creator operators associated with x and p_x in the canonical configuration representation, respectively. I now remark that the fractal operator q^N can be realized in \mathcal{F} as:

$$q^N \psi(\alpha) = \frac{1}{\sqrt{q}} \exp\left(\frac{\zeta}{2}(c^2 - c^{\dagger 2})\right) \psi(\alpha) \equiv \frac{1}{\sqrt{q}} \hat{S}(\zeta) \psi(\alpha) \equiv \frac{1}{\sqrt{q}} \psi_s(\alpha), \quad (30)$$

where $q = e^\zeta$ (for simplicity, assumed to be real) and as usual $N = \alpha \frac{d}{d\alpha}$. $\hat{S}(\zeta)$ is defined to be the squeezing operator well known in quantum optics [44, 55]. $\zeta = \ln q$ is called the squeezing parameter. In (30) $\psi_s(\alpha)$ denotes the squeezed states in FBR. From Eq. (30) we see that q^N acts in \mathcal{F} , as well as in the configuration representation in the limit $y \rightarrow 0$, as the squeezing operator $\hat{S}(\zeta)$, up to the numerical factor $\frac{1}{\sqrt{q}}$.

Since $q^N \psi(\alpha) = \psi(q\alpha)$ (cf. Eq. (22)), from Eq. (30) we see that the q -deformation process, which we have seen is associated to the fractal generation process, is equivalent to the squeezing transformation.

The right hand side of (30) is a $SU(1, 1)$ group element. We indeed obtain the $SU(1, 1)$ Bogoliubov (squeezing) transformations for the c 's operators:

$$\hat{S}^{-1}(\zeta) c \hat{S}(\zeta) = c \cosh \zeta - c^\dagger \sinh \zeta, \quad (31)$$

$$\hat{S}^{-1}(\zeta) c^\dagger \hat{S}(\zeta) = c^\dagger \cosh \zeta - c \sinh \zeta, \quad (32)$$

and in the $y \rightarrow 0$ limit

$$\hat{S}^{-1}(\zeta) \alpha \hat{S}(\zeta) = \frac{1}{q} \alpha \rightarrow \frac{1}{q} x, \quad (33)$$

$$\hat{S}^{-1}(\zeta) p_\alpha \hat{S}(\zeta) = qp_\alpha \rightarrow qp_x, \quad (34)$$

where $p_\alpha \equiv -i \frac{d}{d\alpha}$, and

$$\int d\mu(\alpha) \bar{\psi}(\alpha) \hat{S}^{-1}(\zeta) \alpha \hat{S}(\zeta) \psi(\alpha) \rightarrow \frac{1}{q} \langle x \rangle \quad (35)$$

$$\int d\mu(\alpha) \bar{\psi}(\alpha) \hat{S}^{-1}(\zeta) p_\alpha \hat{S}(\zeta) \psi(\alpha) \rightarrow q \langle p_x \rangle \quad (36)$$

so that the root mean square deviations Δx and Δp_x satisfy

$$\Delta x \Delta p_x = \frac{1}{2}, \quad \Delta x = \frac{1}{q} \sqrt{\frac{1}{2}}, \quad \Delta p_x = q \sqrt{\frac{1}{2}}. \quad (37)$$

This confirms that the q -deformation plays the role of squeezing transformation. Note that the action variable $\int p_x dx$ is invariant under the squeezing transformation.

Eq. (33) shows that $\alpha \rightarrow \frac{1}{q} \alpha$ under squeezing transformation, which, in view of the fact that $q^{-1} = \alpha$ (cf. Eq. (4)), means that $\alpha \rightarrow \alpha^2$, i.e. under squeezing we proceed further in the fractal iteration process. Thus, the fractal iteration process can be described in terms of the coherent state squeezing transformation.

Besides the scale parameter one might also consider, phase parameters and translation parameters characterizing (generalized) coherent states (such as $SU(2)$, $SU(1, 1)$, etc. coherent states). For example, by changing the parameters in a *deterministic iterated function process*, also referred to as *multiple reproduction copy machine* process [50], (such as phases, translations, etc.) the Koch curve may be transformed into another fractal (e.g. into Barnsley's fern [50]). In the scheme here presented, these fractals are described by corresponding unitarily inequivalent representations in the limit of infinitely many degrees of freedom (infinite volume limit) [52]. See Ref. 52 for further details on the functional representation of self-similarity in terms of entire analytic functions.

I conclude that, since the vacuum states in the dissipative many-body model are squeezed coherent states, they provide the functional representation of self-similarity observed in neuro-phenomenological data.

III. TRAJECTORIES IN THE ATTRACTOR LANDSCAPE

Dissipation is a key ingredient which allows to exploit the infinitely many unitarily inequivalent representations of QFT. Each spatial AM pattern is described to be consequent to spontaneous breakdown of symmetry triggered by external stimulus and is associated with one of the unitarily inequivalent ground states. Their sequencing is associated to the non-unitary time evolution implied by dissipation. Changes in the brain-environment interaction produce changes in the brain ground state. The brain evolution through the vacuum states thus reflects the evolution of the coupling of the brain with the surrounding world. In the memory space, i.e. the *brain state space*, each representation $\{|0\rangle_{\mathcal{N}}\}$ denotes a physical phase of the system and may be conceived as a “point” identified by a specific \mathcal{N} -set in a “landscape of attractors”. Vacuum states are indeed least energy states towards which time evolution proceeds and thus they act as dynamical “attractors”. Under the influence of one or more stimuli brain

may undergo an extremely rich sequence of phase transitions, namely a sequence of dissipative structures formed by AM patterns, through trajectories in such landscape of attractors.

These trajectories turn out to be *classical* trajectories [56] which may also be *chaotic* [32, 33] and *itinerant* through a chain of 'attractor ruins' [57, 58, 59, 60, 61]. *The possibility of deriving from the microscopic dynamics the classicality of such trajectories is one of the merits of the dissipative many-body model.*

The entropy, for both a and \tilde{a} system, is found to grow monotonically from 0 to infinity as the time goes to $t = \infty$ [21]. For the complete system $a - \tilde{a}$, the difference $(S_a - S_{\tilde{a}})$ is constant in time. The change in the energy $E_a \equiv \sum_k E_k \mathcal{N}_{a_k}$ and in the entropy is given by

$$dE_a = \sum_k E_k \dot{\mathcal{N}}_{a_k} dt = \frac{1}{\beta} dS_a, \quad (1)$$

so that the free energy \mathcal{F}_a of the brain system is minimized on the trajectories:

$$d\mathcal{F}_a \equiv dE_a - \frac{1}{\beta} dS_a = 0, \quad (2)$$

provided changes in inverse temperature are slow, i.e. $\frac{\partial \beta}{\partial t} = -\frac{1}{k_B T^2} \frac{\partial T}{\partial t} \approx 0$, which is what actually happens in mammalian brains which keep their temperature nearly constant. As usual heat is defined as $dQ = \frac{1}{\beta} dS$. The change in time of condensate ($\dot{\mathcal{N}}_{a_k}$, Eq. (1)) turns out to be heat dissipation dQ . Thus, entropy changes and heat dissipation involved in the disappearance/emergence of the coherence (ordering) associated to the AM patterns turns into energy changes. Heat dissipation is indeed a significant variable in laboratory observations. Brains require constant perfusion with arterial blood and venous removal to dispose of waste heat [30]. I observe that entropy variation of the system also implies variation of the entropy of the environment. The reciprocal system–environment interaction is thus a reciprocal back–reaction process. Then, the process of minimizing the free energy characterizing the system evolution can be thought as a "survival" strategy of the system producing continual adaptation of system to the environment and, at once, continual environmental modifications.

The model predicts condensation domains of different finite sizes with different degrees of stability [31]. Phase transitions driven by boson condensation are associated with some singularity in the field phase at the phase transition point [12, 30, 62, 63]. This specific feature of the model accounts [30] for a crucial mechanism observed in laboratory experiments: the event that initiates a perceptual phase transition is an abrupt decrease in the analytic power of the background activity to near zero (null spike), associated with the concomitant increase of spatial variance of analytic phase. The null spikes recur aperiodically at rates in the theta (3 – 7 Hz) and alpha (8 – 12 Hz) ranges. These null spikes are of crucial importance for the brain–environment interaction. They allow the readiness of the brain to react to an input incoming at or just before the null spike.

The incoming input is "expected" on the basis of the attractor landscape as modified by the brain "experiences" with previously received inputs. In this way, on the basis of these recurrent *rearrangements* of the attractor landscapes, the brain promotes and controls the *action* which is thus *intentionally* guided in order to have the "maximal grip" [64, 65] on the external world. The selective sensitivity to perceptual inputs which better fit the existing attractor landscapes (which specifies in which sense incoming inputs are "expected") is controlled by the process of activation of mesoscopic neural patterns related with such landscapes and is termed preaffference [66, 67].

Moreover, in the (cyclic) process of action–perception, the continual reshaping and rearrangement of the attractor landscapes, due to the introduction in the "memory state space" of the new vacuum condensate triggered by a forthcoming stimulus, constitutes the "contextualization" process by which, by differentiation with preexisting landscape arrangement, a *meaning* [68, 69] is attached to such incoming stimulus. The experience of the brain in its relation with the world in which it is *situated* thus becomes "stored experience", or *knowledge* [11, 12], which generates perspectives, hypothesis, expectations, i.e. the *view of the world* reached by that brain in its experience history [34, 70]. Any subsequent action becomes then a *test* for such a "view of the world", which is trustable and generates "confidence" exactly because it cannot escape from being continuously tested in the un–resting action–perception cycle (the brain is intrinsically and thus permanently open on the world). The process of knowledge as described above thus implies in an essential way the rearrangement of the attractor landscape (contextualization), not just "additions" of new "points" in the landscape.

It is interesting that the picture describing the brain's *to-be-in-the-world* [64, 65], emerging from neuroscience observations and their dissipative model formalization as deeply rooted in the brain un–avoidable experiential dimension, closely depicts Galileo's paradigm for the New Science.

IV. CONCLUSIVE REMARKS. TO-BE-IN-THE-WORLD: I AND MY DOUBLE

As a conclusive remark, let me go back to what said on the environment description at the beginning of Section 2. There we have seen that the environment is described in the dissipative many-body model as the system time-reversed copy, its *Double*. This represents the environment *as seen* from the system. The inter-relation between the system (brain) and the environment is thus the relation of the system with its Double. What we have said in the previous Section may be rephrased in terms of a *dialog* with the Double [21, 34], which is a dynamical one since there is a continuous reciprocal updating, expressing the system's dynamical view of the world. Such a dialog also implies the *emotion* of *novelty* and sometimes the *surprise* or *astonishment* of a freshly acquired *different*, perhaps even *completely different*, view of the world [34, 70, 71]. Remarkably, it has been proposed [72] that the aesthetical experience consists in realizing the perfect accomplishment of our trade with the external world leading us to such an emotion of novelty [70], which at same time was searched and even expected on the basis of our pre-existing landscape scenarios ("education" through the "acquired experiences"). This may shed some light on the relation between the aesthetical experience and Spinoza's "intuitive science" [73].

To-be-in-the-world thus becomes this life "entre-deux", however diffused in the world, the *between* expressing the reciprocal being *exposed* to each other sight, in a continual trade [71]. Consciousness mechanisms may have their roots in this dialog with the Double [21, 34, 71]. Let me then close the paper by reporting the following "thoughts" [74]:

"The other one, the one called Borges, is the one things happen to....It would be an exaggeration to say that ours is a hostile relationship; I live, let myself go on living, so that Borges may contrive his literature, and this literature justifies me....Besides, I am destined to perish, definitively, and only some instant of myself can survive him....Spinoza knew that all things long to persist in their being; the stone eternally wants to be a stone and a tiger a tiger. I shall remain in Borges, not in myself (if it is true that I am someone)....Years ago I tried to free myself from him and went from the mythologies of the suburbs to the games with time and infinity, but those games belong to Borges now and I shall have to imagine other things. Thus my life is a flight and I lose everything and everything belongs to oblivion, or to him.

I do not know which of us has written this page."

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- [1] W. J. Freeman, *Mass Action in the Nervous System* (Academic Press, New York, 1975/2004).
W. J. Freeman, *Neurodynamics. An Exploration of Mesoscopic Brain Dynamics* (Springer-Verlag, London, 2000).
W. J. Freeman, *How Brains Make Up Their Minds* (Columbia University Press, New York, 2001).
 - [2] H. Umezawa, Development in concepts in quantum field theory in half century, *Math. Japonica* **41**(1995) 109–124.
 - [3] L.M. Ricciardi and H. Umezawa, Brain physics and many-body problems, *Kibernetik* **4**(1967) 44–48. Reprint in *Brain and Being*, eds. G. G. Globus, K. H. Pribram and G. Vitiello (John Benjamins, Amsterdam, 2004) pp. 255–266.
C. I. J. Stuart, Y. Takahashi and H. Umezawa, On the stability and non-local properties of memory, *J.Theor. Biol.* **71**(1978) 605–618.
C. I. J. Stuart, Y. Takahashi and H. Umezawa, Mixed system brain dynamics: neural memory as a macroscopic ordered state, *Found. Phys.* **9**(1979) 301–327.
 - [4] K. Lashley, *The Mechanism of Vision, XVIII, Effects of Destroying the Visual "Associative Areas" of the Monkey* (Journal Press, Provincetown MA, 1948).
 - [5] R. J. Glauber, Coherent and Incoherent States of the Radiation Field, *Phys. Rev.* **131**(1963) 2766–2788.
 - [6] J. R. Klauder, and E. C. Sudarshan, *Fundamentals of Quantum Optics* (Benjamin, New York, 1968).
J. R. Klauder and B. Skagerstam, *Coherent States* (World Scientific, Singapore, 1985).
 - [7] D. Gabor, A new Microscopic Principle, *Nature* **161**(1948) 777–778.
 - [8] D. Gabor, Improved holographic model of temporal recall, *Nature* **217**(1968) 1288–1289.
 - [9] K. H. Pribram, *Languages of the Brain* (Prentice-Hall, Engelwood Cliffs, NJ, 1971).
K. H. Pribram, *Brain and Perception* (Lawrence Erlbaum Associates Publ., Hillsdale, NJ, 1991).
 - [10] W. J. Freeman, NDN, volume transmission, and self-organization in brain dynamics, *J. Integrative Neuroscience* **4**(2005) 407–421.
 - [11] W. J. Freeman and G. Vitiello, Nonlinear brain dynamics as macroscopic manifestation of underlying many-body dynamics, *Phys. of Life Reviews* **3**(2006) 93–118, <http://www.arxiv.org/find> [Freeman]; q-bio.OT/0511037.

- [12] W. J. Freeman and G. Vitiello, Brain dynamics, dissipation and spontaneous breakdown of symmetry, *J. Phys. A: Math. Theor.* **41**(2008) 304042, <http://Select.iop.org, q-bio.NC/0701053v1>.
- [13] W. J. Freeman, Origin, structure, and role of background EEG activity. Part 1. Analytic amplitude, *Clin. Neurophysiol.* **115**(2004) 2077–2088.
- [14] W. J. Freeman, Origin, structure, and role of background EEG activity. Part 2. Analytic phase, *Clin. Neurophysiol.* **115**(2004) 2089–2107.
- [15] W. J. Freeman, Origin, structure, and role of background EEG activity. Part 3. Neural frame classification, *Clin. Neurophysiol.* **116** (5)(2005) 1117–1129.
- [16] W. J. Freeman, Origin, structure, and role of background EEG activity. Part 4. Neural frame simulation, *Clin. Neurophysiol.* **117**(3)(2006) 572–589.
- [17] W. J. Freeman, B. C. Burke, M. D. Holmes and S. Vanhatalo, Spatial spectra of scalp EEG and EMG from awake humans, *Clin. Neurophysiol.* **114**(2003) 1055–1060.
- [18] W. J. Freeman, G. Gaál and R. Jorntén, A neurobiological theory of meaning in perception. Part 3. Multiple cortical areas synchronize without loss of local autonomy, *Intern. J. Bifurc. Chaos* **13**(2003) 2845–2856.
- [19] W. J. Freeman and L. J. Rogers, A neurobiological theory of meaning in perception. Part 5. Multicortical patterns of phase modulation in gamma EEG, *Intern. J. Bifurc. Chaos* **13**(2003) 2867–2887.
- [20] D. S. Bassett, A. Meyer-Lindenberg, S. Achard, T. Duke, E. Bullmore, Adaptive reconfiguration of fractal small-world human brain functional network, *Proc. Natl. Acad. Sci. U.S.A.* **103**(2006) 19518–19523.
- [21] G. Vitiello, Dissipation and memory capacity in the quantum brain model, *Int. J. Mod. Phys. B* **9**(1995) 973–989.
- [22] C. Itzykson and J. Zuber, *Quantum field theory* (McGraw-Hill, New York, 1980).
- [23] G. Vitiello, Structure and function. An open letter to Patricia Churchland, in *Toward a science of consciousness II. The second Tucson Discussions and debates*, eds. S.R. Hameroff, A. W. Kaszniak and A.C. Scott (MIT Press, Cambridge, 1998), pp. 231–236.
- [24] H. Umezawa, and G. Vitiello, *Quantum Mechanics* (Bibliopolis, Naples, 1985).
- [25] M. Jibu and K. Yasue, *Quantum brain dynamics and consciousness*, (John Benjamins, Amsterdam, 1995).
- [26] M. Jibu, K. H. Pribram and K. Yasue, From conscious experience to memory storage and retrieval: the role of quantum brain dynamics and boson condensation of evanescent photons, *Int. J. Mod. Phys. B* **10**(1996) 1735–1754.
- [27] E. Del Giudice, S. Doglia, M. Milani and G. Vitiello, A quantum field theoretical approach to the collective behavior of biological systems, *Nucl. Phys. B* **251**(FS 13)(1985) 375–400.
- [28] E. Del Giudice, S. Doglia, M. Milani and G. Vitiello, Electromagnetic field and spontaneous symmetry breakdown in biological matter, *Nucl. Phys. B* **275**(FS 17)(1986) 185–199.
- [29] E. Del Giudice, G. Preparata and G. Vitiello, Water as a free electron laser *Phys. Rev. Lett.* **61**(1988) 1085–1088.
- [30] B. Haider, A. Duque, A. R. Hasenstaub, D. A. McCormick, Neocortical network activity in vivo is generated through a dynamic balance of excitation and inhibition. *J. Neurosci.* **26**(2006) 4535–4545.
- [31] C. C. Petersen, T. T. G. Hahn, M. Mehta, A. Grinvald, B. Sakmann, Interaction of sensory responses with spontaneous depolarization in layer 2/3 barrel cortex, *Proc. Natl. Acad. Sci. U.S.A.* **100**(2003) 13638–13643.
- [32] M. E. Raichle, The Brain's Dark Energy, *Science* **314**(2006) 1249–1250.
- [33] W. J. Freeman and G. Vitiello, Vortices in brain waves, arXiv:0802.3854v1 [q-bio.NC].
- [34] E. Alfinito and G. Vitiello, Formation and life-time of memory domains in the dissipative quantum model of brain, *Int. J. Mod. Phys. B* **14**(2000) 853–868. [Erratum-ibid. **B14**(2000) 1613].
- [35] E. Pessa and G. Vitiello (2003). Quantum noise, entanglement and chaos in the quantum field theory of mind/brain states. *Mind and Matter* **1**(2003) 59–79.
- [36] E. Pessa and G. Vitiello, Quantum noise induced entanglement and chaos in the dissipative quantum model of brain. *Int. J. Mod. Phys. B* **18**(2004) 841–858.
- [37] G. Vitiello, *My Double Unveiled* (John Benjamins, Amsterdam, 2001).
- [38] V. Braitenberg and A. Schüz, *Anatomy of the Cortex: Statistics and Geometry* (Springer-Verlag, Berlin, 1991).
- [39] K. Linkenkaer-Hansen, V. M. Nikouline, J. M. Palva, R. J. Ilmoniemi, Long-range temporal correlations and scaling behavior in human brain oscillations. *J. Neurosci.* **15**(2001) 1370–1377.
- [40] R. C. Hwa and T. Ferree, Scaling properties of fluctuations in the human electroencephalogram. *Phys. Rev. E* **66**(2002) 021901.
- [41] X. F. Wang and G. R. Chen, Complex networks: small-world, scale-free and beyond. *IEEE Circuits Syst.* **31**(2003) 6–20.
- [42] W. J. Freeman, A field-theoretic approach to understanding scale-free neocortical dynamics, *Biol. Cybern.* **92**/6(2005) 350–359.
- [43] P. Franken, A. Malafosse, M. Tafti, Genetic variation in EEG activity during sleep in inbred mice, *Am. J. Physiol.* **275**(1998) R1127–R1137.
- [44] O. I. Lyamin, I. M. Mukhametov, J. M. Siegel, E. A. Nazarenko, I. G. Polyakova, O. V. Shpak, Unihemispheric slow wave sleep and the state of the eyes in a white whale, *Behav. Brain Res.* **129**(2002) 125–129.
- [45] E. Celeghini, M. Rasetti and G. Vitiello, Quantum dissipation, *Annals of Phys.* **215**(1992) 156–170.
- [46] G. Vitiello, Links. Relating different physical systems through the common QFT algebraic structure, in *Quantum Analogues: From Phase Transitions to Black Holes and Cosmology*, eds. W.G. Unruh and R. Schuetzhold, (Lectures Notes in Physics 718) (Springer, Berlin, 2007), pp. 165–205. hep-th/0610094.
- [47] E. Celeghini, S. De Martino, S. De Siena, M. Rasetti and G. Vitiello, Quantum groups, coherent states, squeezing and lattice quantum mechanics, *Ann. Phys.* **241**(1995) 50–67.
- [48] E. Celeghini, M. Rasetti, M. Tarlini and G. Vitiello, $SU(1,1)$ Squeezed States as Damped Oscillators, *Mod. Phys. Lett.*

- B3**(1989) 1213–1220.
- E. Celeghini, M. Rasetti and G. Vitiello, On squeezing and quantum groups, *Phys. Rev. Lett.* **66**(1991) 2056–2059.
- [45] L. C. Biedenharn, and M. A. Lohe, *Quantum Group Symmetry And Q-Tensor Algebras* (World Sc., Singapore 1995).
- D. I. Fivel, Quasi-coherent states and the spectral resolution of the q-Bose field operator, *J. Phys. A: Math. Gen.* **24**(1991) 3575–3586.
- [46] H. Umezawa, *Advanced Field Theory: Micro, Macro and Thermal Physics* (American Institute of Physics, 1993).
- [47] A. Perelomov, *Generalized Coherent States and Their Applications* (Springer-Verlag, Berlin 1986).
- [48] A. Bunde and S. Havlin (Eds.), *Fractals in Science* (Springer -Verlag, Berlin 1995).
- [49] M. I. Stamenov, The fractal-like roots of mind: a tutorial direct access in *Fractals of Brain, Fractals of Mind*, eds. E. Mac Cormac and M. I. Stamenov (John Benjamins Pub. Co. Amsterdam 1996), pp. 273–322.
- [50] H. O. Peitgen, H. Jürgens and D. Saupe, *Chaos and fractals. New frontiers of Science* (Springer-Verlag, Berlin 1986).
- [51] G. Vitiello, Classical trajectories and Quantum Field Theory, *Brazilian J. of Physics* **35**(2005) 351–358.
- [52] G. Vitiello, Topological defects, fractals and the structure of quantum field theory, *E. J. Theor. Phys.*, in print. arXiv:0807.2164v2 [hep-th].
- [53] P. Bak and M. Creutz, Fractals and self-organized criticality, in *Fractals in Science*, eds. A. Bunde and S. Havlin, (Springer -Verlag, Berlin, 1995).
- [54] V. Bargmann, On a Hilbert space of analytic functions and an associated integral *Comm. Pure and Appl. Math.* **14**(1961) 187–214.
- [55] H. P. Yuen, Two-photon coherent states of the radiation field, *Phys. Rev.* **A13**(1976) 2226–2232.
- [56] G. Vitiello, Classical chaotic trajectories in quantum field theory, *Int. J. Mod. Phys.* **B18**(2004) 785–792.
- [57] I. Tsuda, Towards an interpretation of dynamic neural activity in terms of chaotic dynamical systems. *Brain and Behavioral Science* **24**(2001) 793–810.
- [58] C. A. Skarda and W. J. Freeman, How brains make chaos in order to make sense of the world. *Brain and Behavioral Science* **10**(1987) 161–195.
- [59] J. A. S. Kelso, *Dynamic Patterns: The Self Organization of Brain and Behavior* (MIT Press, Cambridge, 1995).
- [60] S.L. Bressler and J.A.S. Kelso, Cortical coordination dynamics and cognition, *Trends Cog. Sci.* **5**(2001) 26–36.
- [61] A. A. Fingelkurts and A. A. Fingelkurts, Operational architectonics of the human brain biopotential field: toward solving the mind-brain problem, *Brain and Mind* **2**(2001) 261–296.
- [62] E. Alfinito, O. Romei and G. Vitiello, On topological defect formation in the process of symmetry breaking phase transitions, *Mod. Phys. Lett.* **B16**(2002) 93.
- [63] E. Alfinito and G. Vitiello, On domain formation in non-instantaneous symmetry breaking phase transitions, *Phys. Rev.* **B65**(2002) 054105.
- [64] M. Merleau-Ponty, *Phenomenology of Perception* (C. Smith, trans.) (Humanities Press, New York, 1945/1962).
- [65] H. L. Dreyfus, Walter Freeman’s Merleau-Pontian Neurodynamics, *Talk delivered at the Conference on Brain Network Dynamics, University of California, Berkeley, January 2007.*
- H.L. Dreyfus, Why Heideggerian AI failed and how fixing it would require making it more Heideggerian, *Philosophical Psychology* **20**[2](2007) 247–248.
- [66] L. M. Kay and W. J. Freeman, Bidirectional processing in the olfactory-limbic axis during olfactory behavior, *Behav Neurosci.* **112**(1998) 541–553.
- [67] R. Kozma, W. J. Freeman, P. Erdí, The KIV model - nonlinear spatio-temporal dynamics of the primordial vertebrate forebrain, *Neurocomputing* **52**(2003) 819–826.
- [68] W. J. Freeman, G. Gaál, R. Jornten, A neurobiological theory of meaning in perception. Part 3. Multiple cortical areas synchronize without loss of local autonomy, *Int. J. Bifurc. Chaos* **13**(2003) 2845–2856.
- [69] W. J. Freeman and L. J. Rogers, A neurobiological theory of meaning in perception. Part 5. Multicortical patterns of phase modulation in gamma EEG, *Int. J. Bifurc. Chaos* **13**(2003) 2867–2887.
- [70] G. Vitiello, Essere nel mondo: Io e il mio Doppio, *Atque* (Firenze), in print
- [71] G. Vitiello, The dissipative brain, in *Brain and Being*, G. G. Globus, K. H. Pribram and G. Vitiello Eds. (John Benjamins, Amsterdam, 2004), pp. 314–334.
- [72] F. Desideri, Il nodo percettivo e la meta-funzionalità dell-estetico, in *Estetiche della percezione*, eds. F. Desideri and G. Matteucci (University Press, Firenze, 2007), pp.13–24.
- [73] R. Diodato, *Vermeer, Góngora, Spinoza. L’estetica come scienza intuitiva* (Bruno Mondatori, Milano, 1997).
- [74] J. L. Borges, Borges and I, in *El hacedor*, Biblioteca Borges, (Alianza Editorial, 1960). English translation in *Labyrinths. Selected Stories and Other Writings*, by J. L. Borges, eds. D. A. Yates and J. E. Irby (New Directions Pu. Co., New York 1964).